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MENGES MODEL APPLIED TO THE ECONOMY OF THE USA

This aim of the article is to investigate whether the particular economic model works in the certain economy. The Menges model was chosen. The country in which the model is tested is the USA. For ensuring the adequacy of the model the complex analysis was applied. The authors determined coefficients significance, the quality and contingency of R^2. It was identified how the variance of regressors explains the variance of regressant and the presence or absence of correlation between them and the conditions of applying the OLS method in order to estimate parameters, the confidence interval and the adequacy of the model. For the visual evidence of correlation the correlation analysis and scatter diagram were developed.

Keywords: Menges model, adequacy of model, *t*-test, R^2-test, *F*-test, GQ-test, DW-test, confidence interval, US economy.

Formulation of the problem generally. The Menges model shows the National Income dependence on National income of previous period and Net Investment. The problem is very important as it explains how the main driver of economy works. The relationship between National Income of current period and National Income of previous period is quite clear as the National Income is the collective value and correlate with the level of life. Investment is the main driver of the economy and adds to the nation's capital stock. It is the most volatile component of GDP. Gross private domestic investment includes the construction of nonresidential structures, the production of equipment and software, private residential construction, and changes in inventories. The bulk of gross private domestic investment goes to the replacement of depreciated capital. Moreover, Investment represents a choice to postpone consumption – it requires saving.

The country which economy we have chosen is the USA. The United States of America is the world's largest national economy, representing 22,4% of nominal global GDP and 16,6% of global GDP (PPP). The United States' GDP was estimated to be $17,555 trillion as of Q3 2014. The U.S. dollar is the currency most used in international transactions and is the world's foremost reserve currency. Several countries use it as their official currency, and in many others it is the de facto currency. The United States has mixed economy and has maintained a stable overall GDP growth rate, a moderate unemployment rate and high levels of research and capital investment. Its six largest trading partners are Canada, China, Mexico, Japan, Germany and South Korea. Total public and private debt was $50,2 trillion at the end of the first quarter of 2010, or 3,5 times GDP. In December 2013, the total of the public debt was about 0,7 times GDP. Domestic financial assets totaled $131 trillion and domestic financial liabilities totaled $106 trillion [1]. This work is based on analysis of the model in the world's largest national economy.

Analysis of recent researches and publications and selection of outstanding issues. There are too small amount of such kind of articles and Menges Model research, that is why this work can be interested in.

The aim of the article is to investigate whether the particular economic model (Menges model) works in the certain economy (the USA's economy).

The main material. ***Specification of the model.*** By the first principle of specification each economic law can be translated into mathematical language. The first step that should be done in the context of this work is the model specification construction [2]. There are several rules, which should be respected: the quantity of equations should be equal to the quantity of endogenous variables (second principle), all variables should be dated (third principle), and the equation should include disturbance term (forth principle). Constructing Menges model:

$$\begin{cases} Y_t = a_0 + a_1 \cdot Y_{t-1} + a_2 \cdot I_t + \varepsilon_t, \\ E(\varepsilon_t) = 0, \\ \sigma(\varepsilon_t) = const, \end{cases} \quad (1)$$

where Y_t – national income – it is endogenous, or depended variable; Y_{t-1} – previous value of national income, lagged exogenous variable; I_t – net investment, exogenous variable; ε_t – disturbance term.

Statistical data collected according to the model covers the period from 1970 to 2012. It is enough to investigate whether the model works in the economy. For the testing if the model is adequate the data of 2013 is taken.

For Y_t I the *GNI* [3] expressed in US $bln was taken, Y_{t-1} is the *GNI* of the previous period expressed in US $bln. *GNI* (gross national income) is the sum of value added by all producers – residents of the nation – plus any product taxes (minus subsidies, not included in output), plus income received from abroad such as employee compensation/property income.

Definition of net investment [4] is the change in capital stock in an economy equals to gross expenditure on fixed capital minus consumption of fixed capital.

Gross capital formation [5] (formerly gross domestic investment) consists of outlays on additions to the fixed assets of the economy plus net changes in the level of inventories. Consumption of fixed capital [6], abbreviated CFC, reflects the decline in the value of fixed assets of enterprises, governments and owners of dwellings in the household sector.

The next step that should be done to investigate the relationship between the variables is the performing of correlation analysis ($Y - GNI_t$, $X_1 - GNI_{t-1}$, X_2 – net investment), Table 1.

Table 1 – **Correlation analysis** [6]

	Y	X_1	X_2
Y	1,00		
X_1	0,999	1,00	
X_2	0,87	0,86	1,00

From the Table 1 it can be seen that $\rho = 0{,}999 > 0{,}75$ and it means there is a strong positive linear relationship between GNI_t and GNI_{t-1}. There is also strong positive linear relationship between GNI_t and GNI_{t-1}, because $\rho = 0{,}87 > 0{,}75$. Two exogenous variables – GNI_{t-1} and net investment are also correlated to each other with the strong positive interrelation as $\rho = 0{,}86 > 0{,}75$.

To get the visual evident of the positive influence of the exogenous variable onto endogenous variable, the scatter diagrams are need to be constructed, Figure 1 and Figure 2.

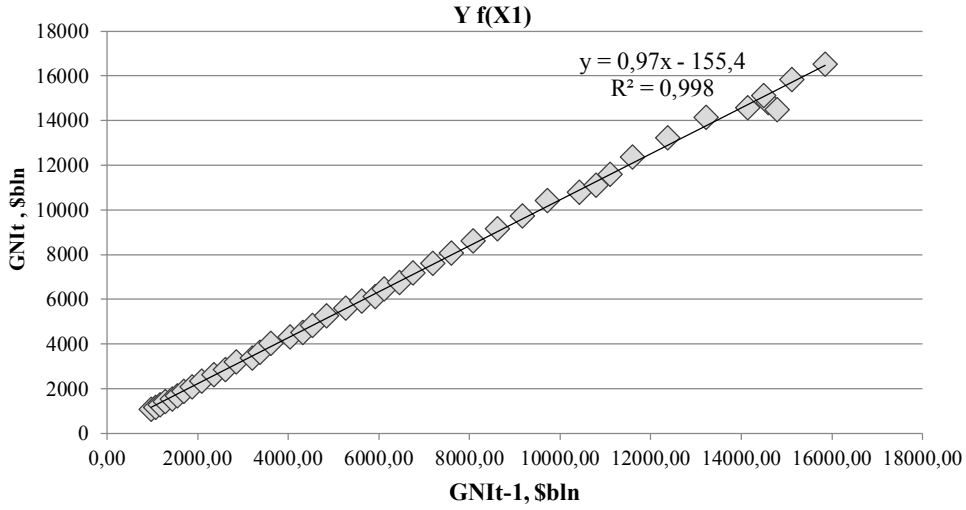

Figure 1 – **Scatter diagram: the dependence National Income on the National Income of the previous period** [6]

$$\begin{cases} y=0,97x-155,4, \\ R^2=0,998. \end{cases} \quad (2)$$

In this model 99,8% of GNI_{t-1} Variance describes the Variance of $GNIt$.

$$\begin{cases} y = 10,62x - 17,24, \\ R^2 = 0,77. \end{cases} \quad (3)$$

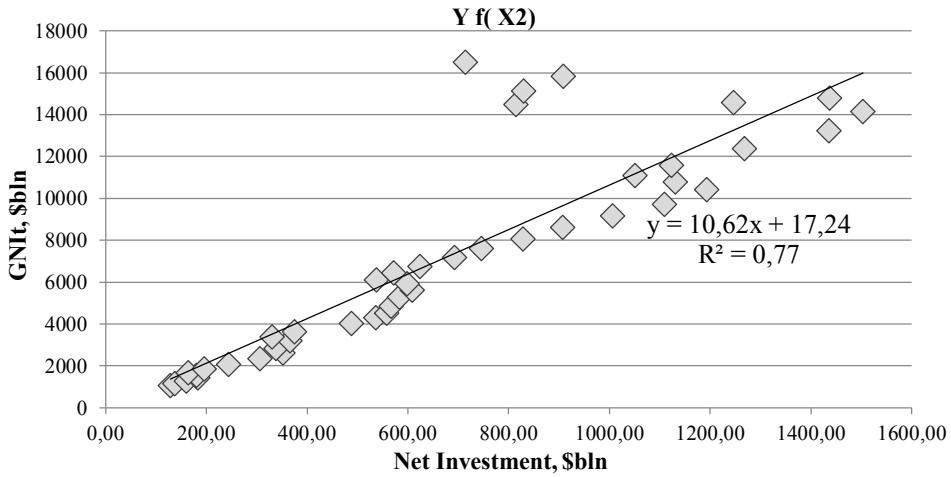

Figure 2 – **Scatter diagram: the dependence National Income on the Net Investment of the previous period** [6]

In this model 77% of GNI_{t-1} Variance describes the Variance of *GNIt*.

Estimated form of the model can be constructed:

$$\begin{cases} Y_t = \underset{(49,25)}{74,13} + \underset{(0,01)}{0,97} \cdot Y_{t-1} + \underset{(0,15)}{0,69} \cdot I_t + \underset{(161,47)}{\varepsilon_t}, \\ \underset{[1,51]}{} \quad \underset{[75,55]}{} \quad \underset{[4,74]}{} \\ R^2=0,999 \quad F=17033,92 \quad F\ crit=3,24 \quad t\ crit=2,71. \\ df_1=39 \quad df_2=2 \end{cases} \tag{4}$$

To estimate the chosen model the regression analysis should be formed. To make regression analysis we use Data Analysis in Excel.

Numbers in the round brackets represent standard deviation of each coefficient. Numbers in square brackets represent *t-statistic*. *df* represents the decrease of freedom, α represents the probability of mistake. $E(\varepsilon_x)$ represents the average mean of residuals. Coefficient 0,97 means that increase of GNI_{t-1} by $US 1 bln will increase GNI_t by $US 0,97 bln. Coefficient 0,69 means that the increase of Net Investment by $US1 bln will increase GNI_t by $US 0,69 bln.

To estimate our model we should carry out few tests. To begin with, we have to make *t*-test to check the significance of chosen coefficients. Using the Excel formula "СТЬЮДРАСПОБР" we calculate *t* critical under probability of mistake 5% and decrease of freedom df_1=39 (Number of observations – number of estimated parameters).

After performing the calculation we get that in our case *t-critical* equals to 2,71. If $|t|>t\ crit$ then coefficient is significant. Now, we compare *t-statistics* and *t-critical*: *t*-statistics for a_0 (1,51) < *t* crit (2,71) – then a_0 is not significant. (*t*-critical was also calculated for probabilities of mistakes equal to 0,01 (2,02) and 0,1 (1,68) – a_0 is not significant anyway):

T-statistics for a_1 *75,55> t crit(2,71)* – it means that a_1 is significant;

T-statistics for a_2 *(4,74) > t crit (2,71)* – it means that a_2 is significant.

The next step is to perform R^2-test. R^2 is equal to 0,999 and it shows that 99% of variation of GNI_t is explained by variation of GNI_{t-1} and I_t, as R^2 leads to 1. In another words the variation of y of the model is well described by y-s of the model.

Finally, the F-test should be made to see whether quality of specification of the chosen model is high or low and whether R^2 is random or not. Using the formula "F.ОБР.ПХ" in Excel F-critical is calculated under probability of mistake 5% and df_2=2 (number of regressors), df_1=39 (number of observations-number of variables). F critical=3,24.

If F *critical* < F then R^2 is not random and quality of specification is high.

F critical (5,14) < F (17033,92) – then R^2 is not random and quality of specification is high.

Goldfeld-Quandt test. To investigate if the residuals are homoscedastic or heteroscedastic the Goldfeld-Quandt test should be done.

What should be known first is whether expectation of residuals is equal to zero. To calculate the average of residuals the formula "СРЗНАЧ" at the table of residual output should be applied, it equals to $E(\varepsilon_x)=-1\cdot10^{-12}$.

Secondly, the initial data should be sorted in ascending order. To make it be possible the absolute value of X_1 *and* X_2 should be calculated and summed (ABS = ABS(X_1) + ABS(X_2)) and then sort data by this parameter. To perform GQ-test data should be divided into two equal parts and then a regression analysis for each part should be done. After that, GQ constant and 1/GQ should be calculated (Dividing SS of the regression analysis of the first part by SS of the regression analysis of the second part) and then compared with F-critical GQ [7].

If GQ < F critical GQ and 1/GQ < F critical GQ then residuals are homoscedastic, if one of conditions does not accomplish-it is heteroscedastic.

$$\begin{cases} GQ\ (0,23) < Fcrit(2,22), \\ \dfrac{1}{GQ}(4,29) > F\ crit\ (2,22). \end{cases} \tag{5}$$

It means, that residuals are heteroscedastic and the method of ordinary square cannot be used in order to estimate parameters of the model. The valuations lose the feature of accuracy, the third condition of Gauss-Markov theorem is not performed [8].

Durbin-Watson test. The next step is to perform Durbin-Watson test to check if there exists correlation between residuals.

If DW is in the interval between 0 and d_l or 4-d_l and 4 then there is a correlation between residuals and the method of OLS cannot be used, the valuation lose the accuracy.

If DW is in the interval between d_l and d_u or 4-d_u and 4-d_l then the valuation is in the ambiguity area and the conclusion about the residuals correlation cannot be made.

If DW is in the interval between d_u and 4-d_u then there is no correlation between residuals.

To calculate DW constant this formula should be used:

$$DW=\frac{\sum(e_t-e_{t-1})^2}{\sum e_t^2}=1,33. \tag{6}$$

The number of parameters equals to 2, the number of observations equals to 41, the level

of significance equals to 5%. Using table of values for Durbin-Watson criteria the values *dl*=1,39 and *du*=1,6 are found. The next step is to construct the interval (Table 2).

Table 2 – **Test of Durbin-Watson under the level of significance equals to 5%** [9]

0	d_l	d_u	2	4-d_u	4-d_l	4
0	1,4	1,6	2	2,4	2,6	4

In this particular case DW (1,33) is in the interval between 0 and d_l then there is a correlation between residuals. This means that the method of OLS cannot be used, the valuation lose the accuracy.

The step that can improve the situation is to widen the interval by changing the level of significance to be equal to 1% (Table 3).

Table 3 – **Test of Durbin-Watson under the level of significance equals to 1%** [9]

0	d_l	d_u	2	4-d_u	4-d_l	4
0	1,2	1,4	2	2,6	2,8	4

In this particular case the valuation is in the ambiguity area and the conclusion about the residuals correlation cannot be made.

Confidence interval and adequacy of the model. Finally, the adequacy of our model should be estimated. For this purpose the adequacy interval should be constructed, by following formula:

$$\left(\hat{Y}\text{-}t_{crit}\cdot\partial;\hat{Y}\text{+}t_{crit}\cdot\partial\right), \tag{7}$$

where $\hat{Y}$ – estimated Y for the next period; *t-critical* – is taken from t-test; σ – standard deviation of the model.

To estimate GNI_t for 2013 we use coefficients a_0, a_1 and a_2 and values X_1 and X_2:

$\hat{Y} = a_0+a_1\cdot X_1+a_2\cdot X_2= 16725{,}81$.

Now confidence interval can be constructed:

$\left(\hat{Y}\text{-}t_{crit}\cdot\partial;\hat{Y}\text{+}t_{crit}\cdot\partial\right)$=(16367,79; 17083,83).

If estimated *GPt* belongs to the confidence then with probability 95% of the model is adequate.

The GNI_{2013}=17057,5; so it is penetrated into confidence interval.

Conclusion and directions of feather researches. After all the research the conclusion that the Menges model works in the economy of US is done.

The model is adequate instead of some tests have not been passed. T-test haven't been passed only for a_0, so it means that only this free member is not significant, F-test have been passed, so it means that R^2 is not random and quality of specification is high. R^2-test showed that variation of *GNI* almost perfectly (99,9%) is described by the variation of regressors. The autocorrelation of residuals or its absence can not be stated because by the test of

Durbin-Watson, it came to be in the area of ambiguity. GQ-test was not passed, it means that the method of OLS can not be used and that residuals are heteroscedastic, the valuations lose the feature of accuracy, the third condition of Gauss-Markov theorem is not performed. It can occur because of some important variable was not included in the model, or because of only one equation of the whole system of Menges equations was taken instead of all the system. Nevertheless, it can be treated if lagged variables will be included. And the statement that the Menges model is suitable for this particular country is the absolute truth because the predicted value satisfy the confidence interval, even taken in account the difficult foreign economic situation dealed with.

1. The economy of USA [Electronic resource]. – Access mode : http://en.wikipedia.org/wiki/Economy_of_the_United_States.
2. Трегуб, А.В. Методика прогнозирования показателей стохастических экономических систем / А.В. Трегуб, И.В. Трегуб // Вестник МГУЛ-Лесной вестник. – 2008. – №2. – С. 144-151.
3. Statistics – GNI – Woldbank. – 2014. – [Electronic resource]. – Access mode : http://data.worldbank.org/indicator/NE.GDI.FTOT.CD.
4. Statistics – Net Investment- Worldbank. – 2014. – [Electronic resource]. – Access mode : http://data.worldbank.org/indicator/NY.ADJ.DKAP.CD.
5. The calculation is performed by Turkina Sofia in "Excel-file "Menges model applied to US economy" on the base of information from the worldbank.org. – Moscow, 2014.
6. Excel-file "Menges model applied to US economy" performed by Turkina Sofia on the base of information from Worldbank.org. – Moscow, 2014.
7. McGraw-Hill. Essentials of Econometrics. Testing a model by a Durbin-Watson test / McGraw-Hill. – 3rd International Edition. – United States Military Academy, West Point, 2006.
8. Shanchenko N.I. Lectures on econometrics : a textbook for university students enrolled in the specialty "Applied informatics (in economics)" / N.I. Shanchenko. – Ulyanovsk : UlSTU, 2008. – 72 p.
9. Excel-file performed by Turkina Sofia on the base of info of Durbin-Wattson table and regression analysis in excel file "Menges model applied to US economy". – Moscow, 2014.
10. Трегуб И.В. Анализ современного состояния и перспективы развития рынка телекоммуникаций / И.В. Трегуб // Телекоммуникации. – 2008. – №10. – С. 40-43.

1. The economy of USA. (n.d.). *en.wikipedia.org*. Retrieved from http://en.wikipedia.org/wiki/Economy_of_the_United_States [in English].
2. Tregub, A.V., & Tregub, I.V. (2008). Method of forecasting the performance of a economical stochastic system. *Vestnik Moskovskogo Gosudarstvennogo Universiteta Lesa – Lestnoy Vestnik*. Retrieved from e-library- http://elibrary.ru/item.asp?id=10331797 [in Russian].
3. Statistics – GNI – Woldbank. – 2014. *data.worldbank.org*. Retrieved from http://data.worldbank.org/indicator/NE.GDI.FTOT.CD [in English].
4. Statistics – Net Investment- Worldbank. – 2014. *data.worldbank.org*. Retrieved from http://data.worldbank.org/indicator/NY.ADJ.DKAP.CD [in English].
5. The calculation is performed by Turkina Sofia in "Excel-file "Menges model applied to US economy" on the base of information from the *Worldbank.org*. Moscow, 2014 [in English].
6. Excel-file "Menges model applied to US economy" performed by Turkina Sofia on the base of information from *Worldbank.org*. Moscow, 2014 [in English].
7. McGraw-Hill. (2006). *Essentials of Econometrics. Testing a model by a Durbin-Watson test.* United States Military Academy, West Point [in English].
8. Shanchenko, N.I. (2008). *Lectures on econometrics.* Ulyanovsk: UlSTU.
9. Excel-file performed by Turkina Sofia on the base of info of Durbin-Wattson table and regression analysis in excel file "Menges model applied to US economy". Moscow, 2014 [in English].

10. Trehub, I.V. (2008). Analiz sovremennoho sostoianiia y perspektivy razvitiia rynka telekommunykatsyi [Analysis of current state and prospects of development of the telecommunications market]. *Telekommunikatsii - Telecommunications, 10,* 40-43 [in Russian].

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Модель Менгеса на прикладі економіки США

Мета статті полягає в дослідженні того, чи працює конкретна економічна модель в умовах певної економіки. Для цього була обрана модель Менгеса. Країна, у якій перевіряється модель – США. Для забезпечення адекватності моделі був застосований комплексний аналіз. У рамках дослідження автори визначають коефіцієнти значущості, якості та випадковості показника R^2. Було встановлено, як різниця регресорів пояснює дисперсію регресанта і присутність або відсутність кореляції між ними. Встановлено, за яких умов застосування методу найменших квадратів для оцінки параметрів є доречним. Визначено довірчий інтервал, за якого модель є адекватною. Для візуального підтвердження одержаних результатів було здіснено кореляційний аналіз, та побудовано діаграму розсіювання.

Ключові слова: модель Менгеса, адекватність моделі, *t*-тест, R^2-тест, *F*-тест, GQ-тест, DW-тест, довірчий інтервал, економіка США.

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Модель Менгеса на примере экономики США

Цель статьи заключается в исследовании того, работает ли конкретная экономическая модель в условиях определенной экономики. Для этого была выбрана модель Менгеса. Страна, в которой проверяется модель – США. Для обеспечения адекватности модели был использован комплексный анализ. В рамках исследования авторы определяют коэффициенты значимости, качества и случайности показателя R^2. Было установлено, как разница регрессоров объясняет дисперсию регрессанта и присутствие или отсутствие корреляции между ними. Определено, при каких условиях использование метода наименьших квадратов для оценки параметров есть уместным. Рассчитан доверительный интервал, при котором модель является адекватной. Для визуального подтверждения полученных результатов был проведен корреляционный анализ, и построено диаграмму рассеивания.

Ключевые слова: модель Менгеса, адекватность модели, *t*-тест, R^2-тест, *F*-тест, GQ-тест, DW-тест, доверительный интервал, экономика США.

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